Performance Assessment Task

Boxing the Pots
Grade 3

The task challenges a student to demonstrate understanding of concepts involved in multiplication and division. A student must make sense of and complete an area model (or array) of objects in rows and columns. A student must understand the size of a single object is its diameter and then determine the number of objects that can be arranged across a given distance. A student must determine the entire number of objects that covers the array.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide.

3.OA.6 Understand division as an unknown-factor problem.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (square cm, square m, square in, square ft, and improvised units).

3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a x b and a x c. Use area models to represent the distributive property in mathematical reasoning.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or

they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Assessment Results

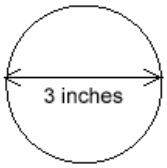
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
3	2004	8	3	61%

Boxing the Pots

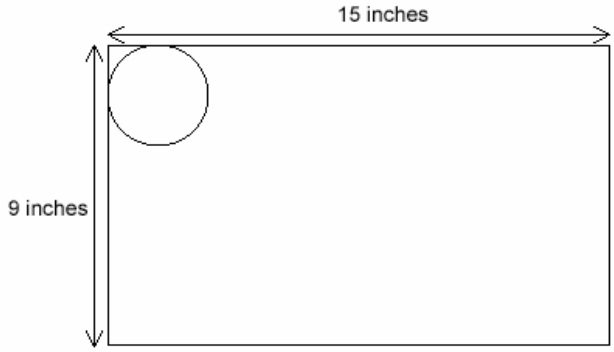
This problem gives you the chance to:
• select and use operations in a real life situation
• use customary units of measurement

At the garden center, Mr. Garcia is putting plant pots into boxes ready to take to market.



The diameter of each plant pot is 3 inches.

Each box measures 9 inches by 15 inches



1. How many pots can Mr. Garcia arrange along the side of the box that measures 15 inches?

   _____

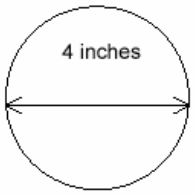
2. How many pots can Mr. Garcia arrange along the side of the box that measures 9 inches?

   _____

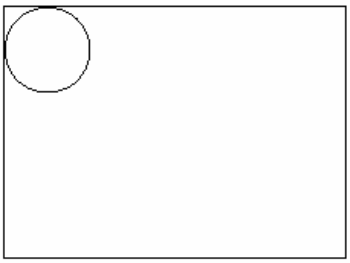
3. How many pots will the box hold? _____

   Show how you figured this out.

4. Mr. Garcia has 12 pots that are 4 inches in diameter.



They can fit exactly into a different box.



What do you think are the measurements of this box?

Length _____ inches

Width _____ inches

Explain how you figured this out.

_____

_____

_____

_____

Boxing the Pots	Test 3 Rubric	
The core elements of performance required by this task are: • select and use operations in a real life situation • use customary units of measurement Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **5**	1	1
2. Gives correct answer: **3**	1	1
3. Gives correct answer: **15** Shows 5 x 3. Accept repeated addition. **or** Draws a diagram showing pots correctly placed in box.	1 ft 1 ft **or** 1 ft	2
4. Gives correct answers: Length = **16** inches **or 24** inches **or 48** inches Width = **12** inches **or 8** inches **or 4** inches Explains the shape of the array. e.g., 4 x 3 pots **or** 6 x 2 pots **or** 12 x 1 pot. Shows how you calculate at least one of the dimensions of the array.	 1 1 1 1	4
Total Points		**8**

Student A shows a good understanding of how to make an array. The student understands that the pot is a measurement unit of 3. This is illustrated in both diagrams. The explanation of part 4 also connects the diagram, pots, and how the measurement units are accumulated to determine the dimensions.

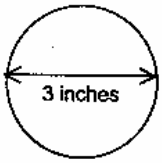
Student A

Boxing the Pots

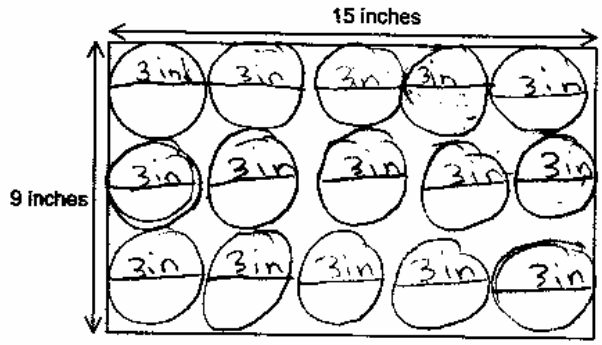
This problem gives you the chance to:
- select and use operations in a real life situation
- use customary units of measurement

At the garden center, Mr. Garcia is putting plant pots into boxes ready to take to market.

The diameter of each plant pot is 3 inches.



Each box measures 9 inches by 15 inches



1. How many pots can Mr. Garcia arrange along the side of the box that measures 15 inches?

   5 ✓ 1

2. How many pots can Mr. Garcia arrange along the side of the box that measures 9 inches?

   3 ✓ 1

3. How many pots will the box hold?

   15 ✓ 1

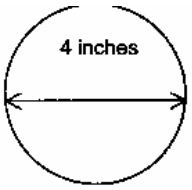
   Show how you figured this out.

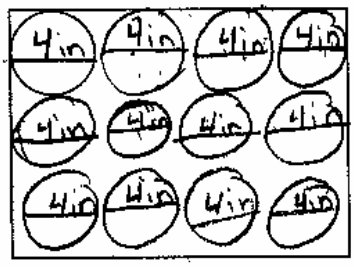
   3×5=15 ✓ 1

Student A

4. Mr. Garcia has 12 pots that are 4 inches in diameter.

They can fit exactly into a different box.





What do you think are the measurements of this box?

Length 16 inches ✓

Width 12 inches ✓

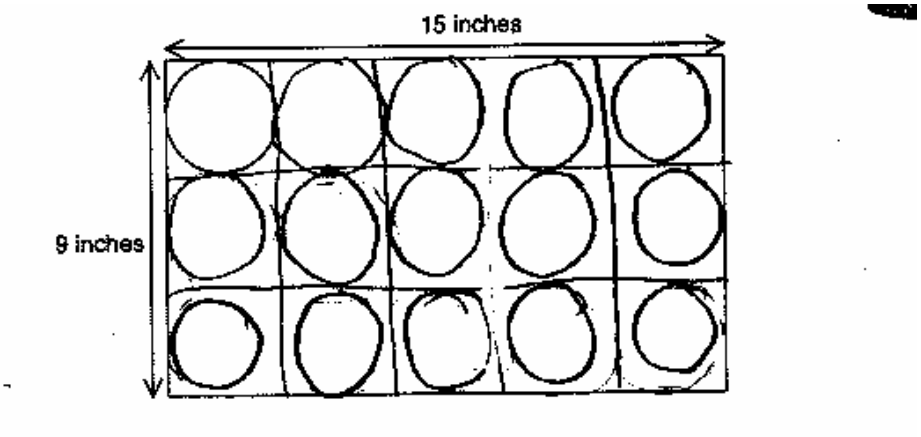
Explain how you figured this out.

I figured it out by drawing in the circles and I multiplied the length with the four circles that fit in the length. It equaled 16 so I knew the length was 16. I did the width the same way but it didn't equal 16 it equal 12. ✓

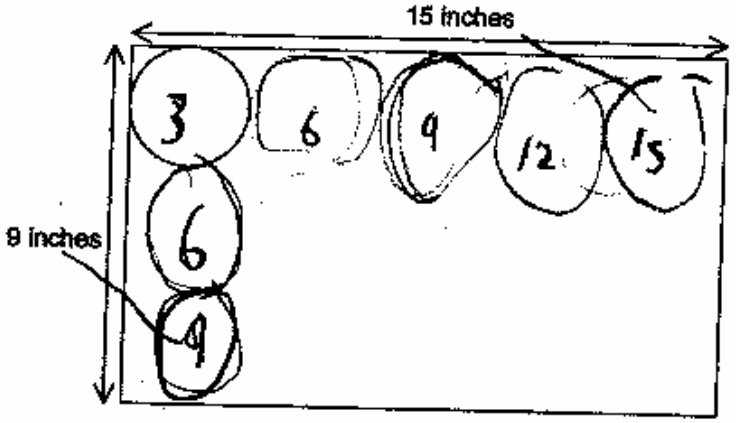
1 1 1 1 1

Most students demonstrated an understanding of how arrays are made of equal size rows and columns. Student B uses a grid to keep the pots aligned. Some students could use just a diagram of the side and top views without filling in the whole array. Student C counts by 3 to know when he has enough pots to match the given dimensions of the box. Student C can then use the number of circles along each edge to calculate the total pots without filling in the whole array.

Student B



Student C



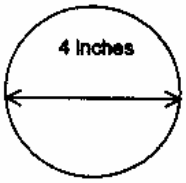
Teacher Notes:

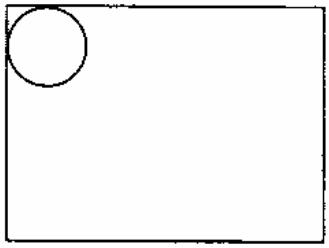
Student D is able to make sense of the relationships of total pots to how the pots are arranged in an array without drawing and can calculate from ports to total dimensions of the box with a clear explanation.

Student D

4. Mr. Garcia has 12 pots that are 4 inches in diameter.

They can fit exactly into a different box.





What do you think are the measurements of this box?

Length 12 ✓ inches

Width 16 ✓ inches

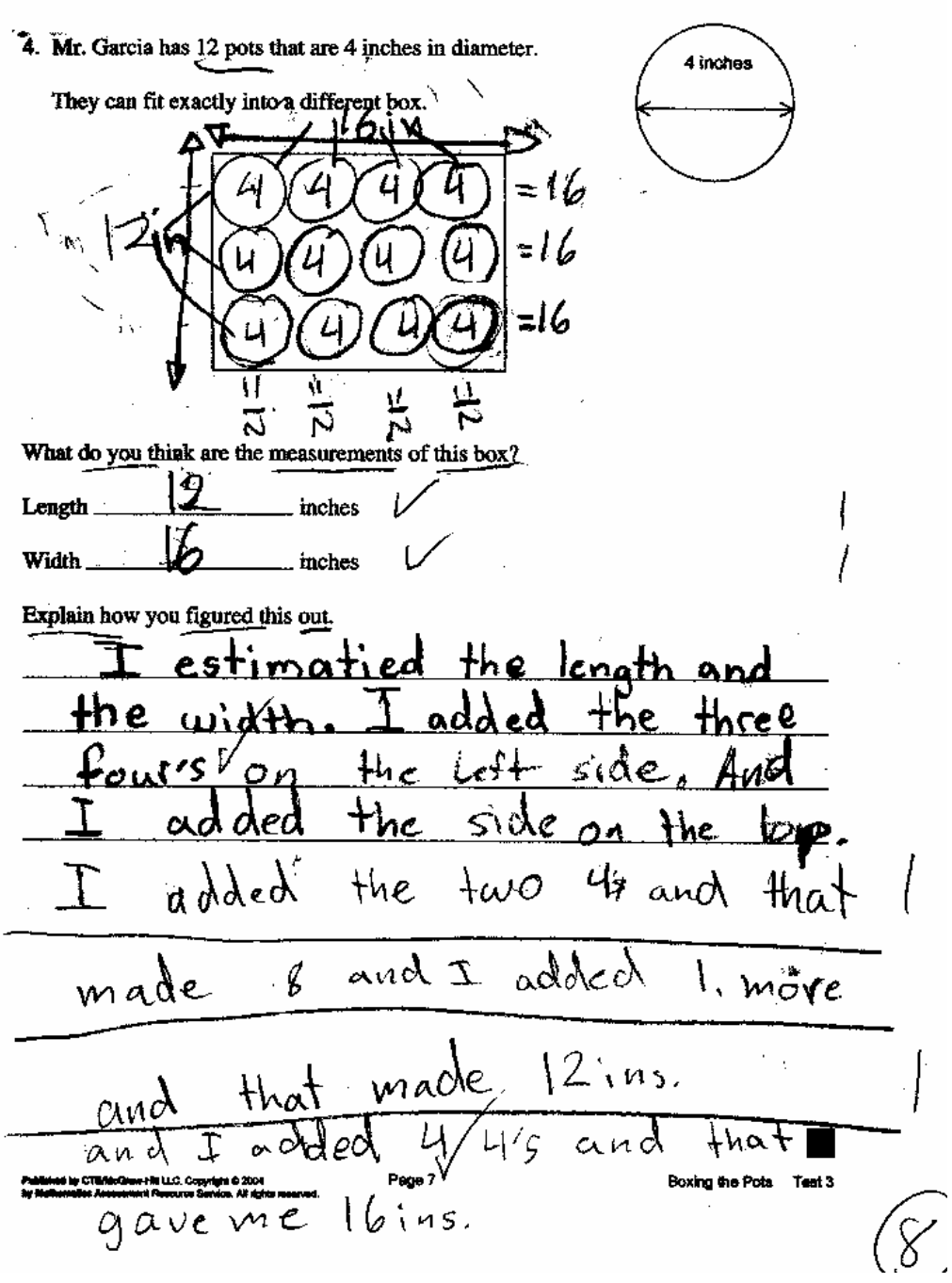
Explain how you figured this out.

4 x 3 = 12 pots ✓ So you need to multiple 4x4 ✓ = 16 and 3x4 = 12 So 16 in width and 12 in length

Teacher Notes:

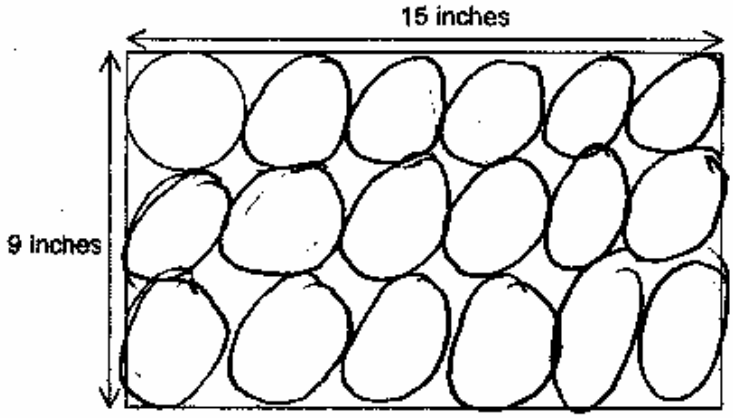
Student E uses the diagram to show how the pots form a 3 by 4 array with equal lengths of rows and columns. The student shows a clear understanding of the pot as a measuring tool.

Student E



While many students understand the idea of the how to fill an array with equal size rows and equal size columns, some students did not understand how to use the dimensions to find the size of the row or column. Student F just comes up with a row of 6 because of the small size of her circles.

Student F

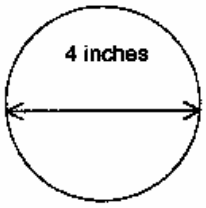


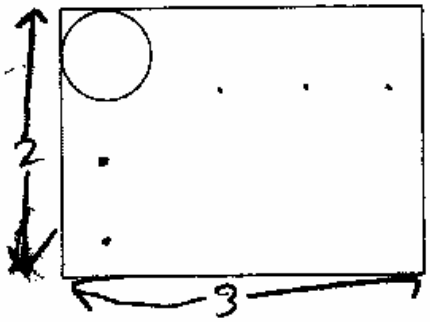
Student G has the idea of an array and how many circles will fit across and down. However the student forgets to count the given circle. The student cannot use the number of pots to calculate the dimensions in inches.

Student G

4. Mr. Garcia has 12 pots that are 4 inches in diameter.

They can fit exactly into a different box.





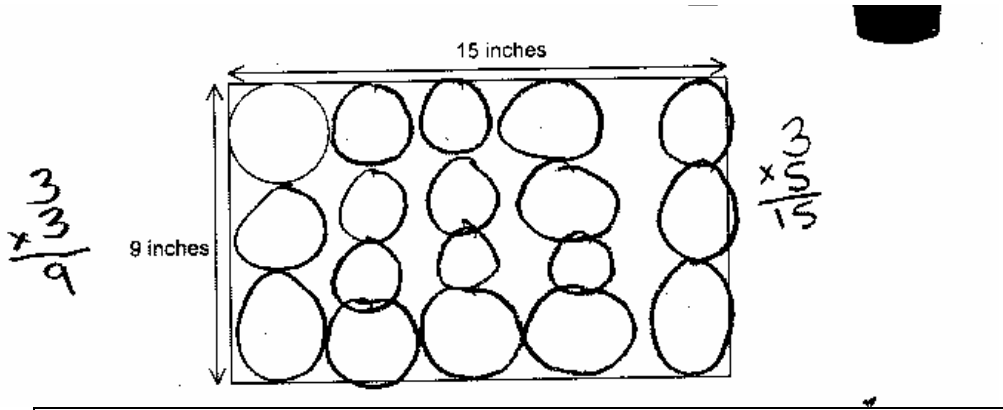
What do you think are the measurements of this box?

Length 3 ✓ inches

Width 2 ✓ inches

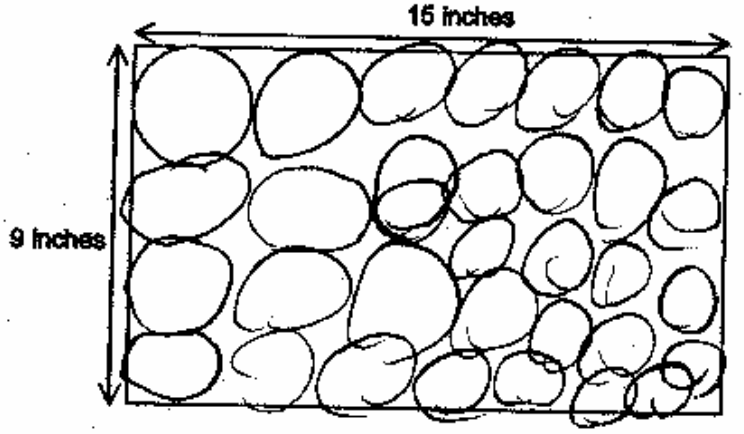
Student H can use the dimensions of the box to find the number of pots needed across the top and along the side. The student fills in extra pots in the middle. The student does not have the idea of how an array is composed of rows, which is a spatial visualization concept that needs to be developed.

Student H



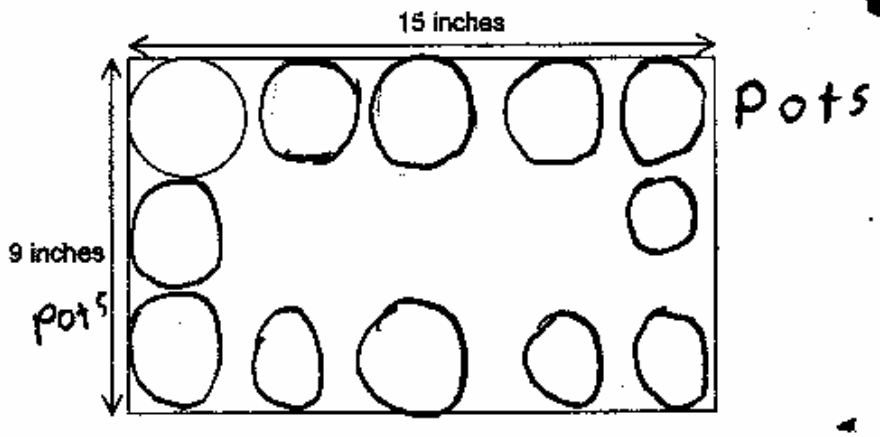
Being able to see the rows and columns in an array is a spatial visualization skill that students need to practice. Student I demonstrates that lack of "rowness" needed to visualize an array. The student does not relate the size of the pot to either dimension of the box.

Student I

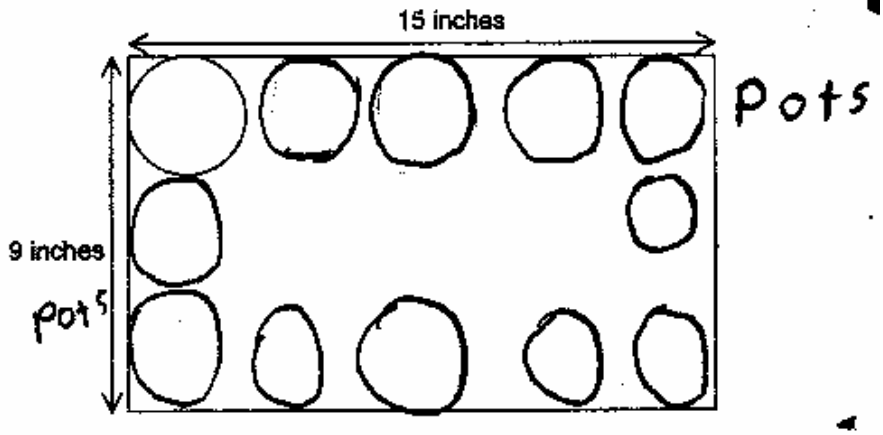


Some students confused total number of pots in the box with the perimeter. Student J draws a picture of pots filling the perimeter of the box and coming up with a total of only 12 pots. Student K shows the division to find clearly the number of pots that would fit in the top and the number of pots that would fit along the side. However the student then thinks that the box has a top and bottom row and 2 sides and therefore calculates 16 total pots. This method is still calculating area, but double counts the 4 corner pots.

Student J

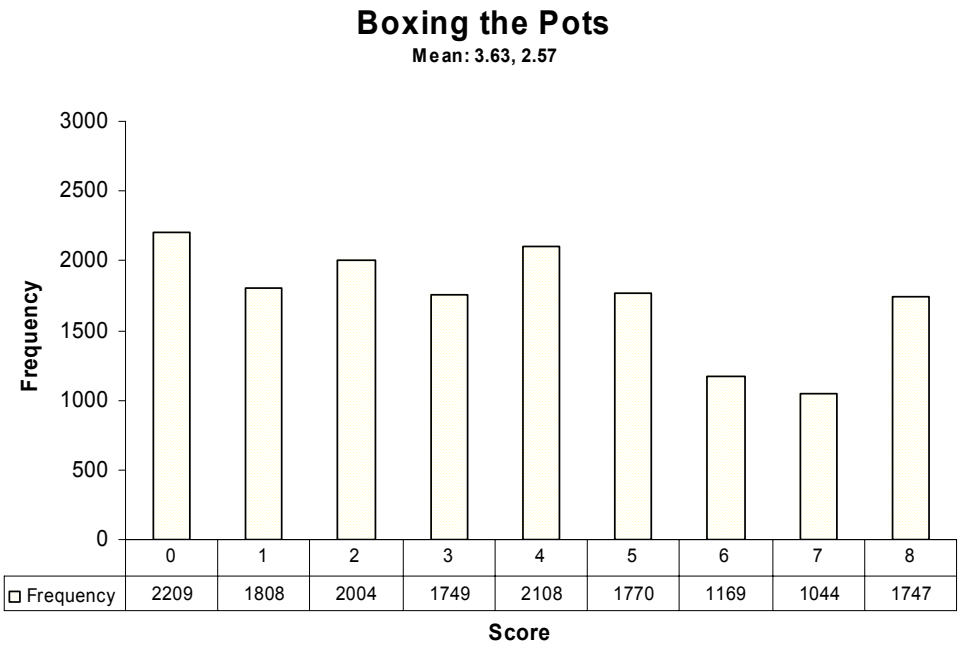


Student K



Teacher Notes:

Frequency Distribution for each Task – Grade 3
Grade 3 – Boxing the Pots



Score:	0	1	2	3	4	5	6	7	8
% < =	14.2%	25.7%	38.6%	49.8%	63.3%	74.6%	82.1%	88.8%	100.0%
% > =	100.0%	85.8%	74.3%	61.4%	50.2%	36.7%	25.4%	17.9%	36.7%

The maximum score available for this task is 8 points.
The cut score for a level 3 response, meeting standards, was 3 points.

Most students (85%) could find the number of pots for at least one dimension in the first box. Many students (61%) could find either both the dimensions and the total number of pots or they found one correct dimension and could use the incorrect dimensions to find the total number of pots. A little more than half the students could find the number of the first box. 12% of the students could meet all the demands of the task, including working backwards from total pots, to how those pots could be arranged in an array, and finally using the dimensions of the pots to find the dimensions of the box. 14% of the students scored no points on this task. About 80% of students with this score attempted the task.

Boxing Pots

Points	Understandings	Misunderstandings
0	80% of the students with this score attempted the task.	Students did not understand arrays and how they are formed. They did not see a connection between the size of the circle and the number of circles needed to fill across the top or along the side of the box.
2	Students could find the number of pots to fit along the top and down the side of the box.	Students could not make the connection between the size of the pot and the dimensions of the box. Students often drew small circles and ending up with 6 or 7 pots across the top of the box.
3	Students may have found an incorrect number of pots across the top of the box, but used this number in a correct process for finding the total number of pots.	Students may not have known how to fill the array, maybe adding in extra circles in the middle, or they may have found the perimeter of pots to fit in the box.
5	Students could find the number of pots to fit along the top and side of the box and use this information to explain the total pots in the box. They could usually find the number of pots to fill each side of a box, given the total number in the box.	Students had difficulty working from the number of pots to the size of the actual box. They could use the information on size of pots and number of pots per side to find length and width of the box. Many students found that they could draw in 4 by 3 pots or 4 by 4 pots and thought that was also the dimensions for the box. Others measured the actual picture and put answers like 2 1/2 inches by 2 inches or 3 inches by 2 inches.
7	Students could use the dimensions to find the number of pots on each side of the box and total number of pots in a box. They could work backwards from the total pots to find one array for making the box and using the size of the pot give the correct dimensions of the box.	Students had difficulty explaining their thinking about working from total pots to dimensions. They were fairly good about explaining how they new the number of pots to fit on the top and bottom of the box. They generally did not articulate how they changed from number of pots to number of inches.
8	Students had the spatial skills to see rows and columns. They could fill in an array and think about how dimensions affected the number of pots and also think about how number of pots changed or effected length and width. They were also able to document how they thought about this process.	

Based on teacher observations, this is what third grade students seemed to know and be able to do:

- Knew division and multiplication facts
- Understood concept of array and could draw pictures to show how pots would fill a box

Areas of difficulty for third graders, third graders struggled with:

- Working backwards from total number of pots to finding an array to yield that number of pots
- Working from the width of the pots to the width or length of the box
- Working with scale drawings
- Using a measurement unit that isn't one

Questions for Reflection on Boxing Pots

- What types of measurement opportunities have your students had this year? Can they work with a measurement unit different than one (in this problem they had to work with a unit of 3 and a unit of 4)?
- What evidence did you have that students could see and understand the structure of an array? Did they add extras in the middle or leave out some of the circles in the middle?
- Look at how students figured out their answers to part 3. How many students:

Used a drawing of the array and multiplication	Used only a drawing	Used only multiplication	Showed division to find #of pots per side (15/3)	Did something with perimeter instead of total	Other

Now look at the answers to part 3 a second time to look at their understanding of how arrays are formed:

Correctly fill the array	Add extras to the array	Attempts to fill in array but leaving some out	Correctly fills an array but used the wrong number pots for one or both directions	Only puts circles around the perimeter	Only puts pots across top and along the side	Other

Many students had trouble working backwards from the total number of pots to how the pots should be arranged or confused the arrangement of the pots with the dimensions.

- Is there some evidence of your students knowing how the pots should be arranged to make a total of 12? How did you know, what convinced you?
- Did your students attempt to use the diameter of the circle, 4 inches, to find the dimensions of the sides? What did successful students do to solve this part of the task?

Teacher Notes:

Implications for Instruction:

Students need help developing spatial visualization skills, so they develop the ability to see rows and columns that form arrays. Students move from filling in spaces randomly or in circles to gradually working in rows and columns. This comes from having many experiences looking at and describing shapes and having opportunities to draw and fill in shapes. Students need help understanding multiplication as an operation for working with equal size groups. This understanding of the equal groups allows them to think about how many groups of "3-inch" or "4-inch" pots fit along a side of a shape. They develop the ability to measure in groups of 3, 5, etc. This multiplicative thinking builds the foundation necessary for comprehending ratios and proportions at later grades. Students at this grade level need to learn and be facile with multiplication facts and fact families. This knowledge allows them to think backward from the total boxes to a fact family that will give that answer.